

# Simulation of 3-D Nonequilibrium Seeded Airflow in the NASA Ames MHD Channel

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The 3-D nonequilibrium seeded airflow in the NASA Ames experimental magnetohydrodynamics channel has been numerically simulated. The channel contains a nozzle section, a center section, and an accelerator section in which magnetic and electric fields can be imposed on the flow. In recent tests, velocity increases of up to 40% have been achieved in the accelerator section. The flow in the channel is numerically computed using a 3-D parabolized Navier–Stokes algorithm that has been developed to efficiently compute magnetohydrodynamics flows in the low magnetic Reynolds number regime. The magnetohydrodynamics effects are modeled by introducing source terms into the parabolized Navier–Stokes equations, which can then be solved in a very efficient manner. The algorithm has been extended in the present study to account for nonequilibrium seeded airflows. The electrical conductivity of the flow is determined using the program of Park. The new algorithm has been used to compute two test cases that match the experimental conditions. In both cases, magnetic and electric fields are applied to the seeded flow. The computed results are in good agreement with the experimental data.

## Nomenclature

<b>B</b>	= magnetic field vector
$B_x, B_y, B_z$	= Cartesian components of magnetic field vector
$C_{p,s}$	= species-specific heat at constant pressure
$c_s$	= species mass fraction
$D$	= binary diffusion coefficient
<b>E</b>	= electric field vector
$\mathbf{E}_i, \mathbf{F}_i, \mathbf{G}_i$	= inviscid flux vectors
$\mathbf{E}_v, \mathbf{F}_v, \mathbf{G}_v$	= viscous flux vectors
$E_x, E_y, E_z$	= Cartesian components of electric field vector
$e$	= static internal energy
$e_t$	= total internal energy
$h_s$	= species enthalpy
<b>I</b>	= identity tensor
$i$	= spatial index in marching direction
<b>J</b>	= conduction current density vector
$J$	= Jacobian of transformation
$K_{f,k}, K_{b,k}$	= forward and backward reaction rates for reaction $k$
$k$	= coefficient of thermal conductivity
$L$	= reference length
$m$	= total number of reactions
$n$	= total number of species
$p$	= static pressure
<b>Q</b>	= heat flux vector

$Re_m$	= magnetic Reynolds number
$Re_\infty$	= Reynolds number ( $\rho_\infty V_\infty L / \mu_\infty$ )
$S_{\text{MHD}}$	= MHD source term
$s$	= species number
$T$	= static temperature
$t$	= time
<b>U</b>	= flux vector of conserved variables
$u, v, w$	= Cartesian components of velocity vector
<b>V</b>	= velocity vector
$V_\infty$	= magnitude of freestream velocity
$x, y, z$	= Cartesian coordinates
$\beta_3$	= $\rho_\infty D_\infty / \mu_\infty$
$\gamma$	= ratio of specific heats
$\tilde{\gamma}$	= effective $\gamma$
$\mu$	= coefficient of viscosity
$\mu_e$	= magnetic permeability
$\xi, \eta, \zeta$	= generalized coordinates
$\rho$	= density
$\sigma_e$	= electrical conductivity
$\bar{\tau}$	= shear stress tensor
$\omega$	= Vigneron parameter
$\dot{\omega}_s$	= species production term

## Subscripts

$i$	= spatial index in marching direction
$o$	= stagnation condition
$t$	= derivative with respect to time
$w$	= wall condition
$x, y, z$	= derivative with respect to $x, y, z$
$\xi, \eta, \zeta$	= derivative with respect to $\xi, \eta, \zeta$
$\infty$	= freestream condition

## Superscripts

$k$	= iterative or sweep index
$*$	= nondimensional quantity
$'$	= streamwise viscous flow terms are omitted

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## Introduction

**M**AGNETOHYDRODYNAMICS (MHD) can be used to improve performance and extend the operational range of many systems. Potential applications include hypersonic cruise, advanced Earth-to-orbit propulsion, chemical and nuclear space propulsion, regenerative aerobraking, onboard flow control systems, test facilities, launch assist, and power generation. One of the critical technologies associated with these applications is MHD acceleration. To study MHD acceleration, an experimental MHD channel has been built at the NASA Ames Research Center by Bogdanoff et al. [1,2]. The channel is about a half-meter long and contains a nozzle section, a center section, and an accelerator section. A cross section of the MHD channel is shown in Fig. 1. The channel has a uniform width of 2.03 cm. Magnetic and electric fields can be imposed upon the flow in the accelerator section. The accelerator section is the portion of the MHD channel (see Fig. 1) at which the electrodes are located. A schematic of the accelerator section (i.e., powered portion) of the channel is shown in Fig. 2.

In the present study, the flow in the experimental MHD channel is numerically simulated. Flowfields involving MHD effects have typically been computed [3–15] by solving the complete Navier–Stokes (N–S) equations for fluid flow in conjunction with Maxwell’s equations of electromagnetodynamics. When chemistry and turbulence effects are also included, the computational effort required to solve the resulting coupled system of partial differential equations is extremely formidable. One possible remedy to this problem is to use the parabolized Navier–Stokes (PNS) equations in place of the N–S equations. The PNS equations can be used to compute three-dimensional, supersonic viscous flowfields in a very efficient manner [16]. This efficiency is achieved because the equations can be solved using a space-marching technique, as opposed to the time-marching technique that is normally employed for the complete N–S equations.

Recently, the present authors have developed PNS codes to solve 2-D and 3-D supersonic MHD flowfields in both the high and low

magnetic Reynolds number regimes [17–19]. The magnetic Reynolds number is defined as  $Re_m = \sigma_e \mu_e V_\infty L$ . The new MHD PNS codes are based on NASA’s upwind PNS (UPS) code, which was originally developed by Lawrence et al. [20]. The UPS code solves the PNS equations using Roe’s scheme in a fully conservative, finite volume approach in general nonorthogonal coordinates.

For many aerospace applications, including the present experimental MHD channel, the electrical conductivity of the fluid is low and hence the magnetic Reynolds number is small. In these cases, it makes sense to use the low magnetic Reynolds number assumption and reduce the complexity of the governing equations. The MHD effects are modeled with the introduction of source terms into the PNS equations.

Previously [19], the present authors used the low magnetic Reynolds number PNS code to compute both 2-D and 3-D flows in the NASA Ames MHD channel. These perfect gas ( $\tilde{\gamma} = 1.25$ ) calculations assumed that the magnetic and electric fields, as well as the electrical conductivity, were constant in the accelerator section. In the present study, the 3-D simulations have been extended to include both equilibrium air flows and nonequilibrium seeded airflows. For the equilibrium air flow computations, the effects of seeding are simulated by using equilibrium clean-air chemistry along with a constant electrical conductivity. For the nonequilibrium seeded-air calculations, a complete chemistry model that includes a potassium-seeding reaction is solved to find the species mass fractions. The electrical conductivity is then computed at each point in the flowfield using the program of Park [21].

## Governing Equations

### Magnetogasdynamics Equations

The governing equations for a viscous MHD flow with a small magnetic Reynolds number are given by [14] continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (1)$$

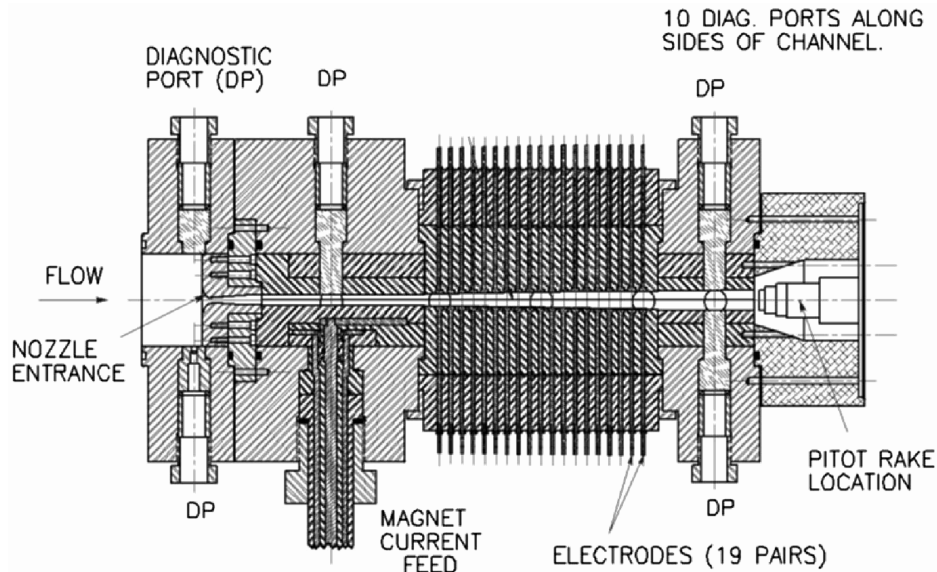


Fig. 1 Cross section of the NASA Ames MHD channel.

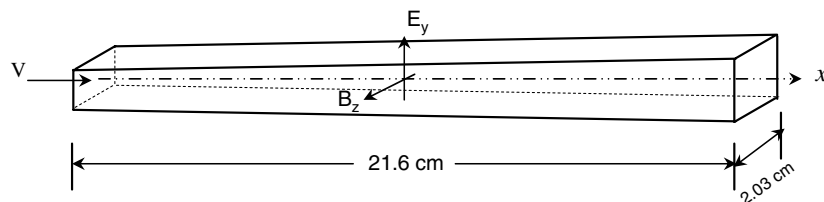


Fig. 2 Schematic of the powered portion of the MHD channel.

momentum equation

$$\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla \cdot [\rho \mathbf{V} \mathbf{V} + p \bar{\mathbf{I}}] = \nabla \cdot \bar{\bar{\tau}} + \mathbf{J} \times \mathbf{B} \quad (2)$$

energy equation

$$\frac{\partial(\rho e_t)}{\partial t} + \nabla \cdot [(\rho e_t + p) \mathbf{V}] = \nabla \cdot (\mathbf{V} \cdot \bar{\bar{\tau}}) - \nabla \cdot \mathbf{Q} + \mathbf{E} \cdot \mathbf{J} \quad (3)$$

Ohm's law

$$\mathbf{J} = \sigma_e (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \quad (4)$$

The governing magnetogasdynamic equations are nondimensionalized using the following reference variables:

$$\begin{aligned} x^*, y^*, z^* &= \frac{x, y, z}{L}, & u^*, v^*, w^* &= \frac{u, v, w}{V_\infty}, & t^* &= \frac{V_\infty t}{L} \\ \rho^* &= \frac{\rho}{\rho_\infty}, & T^* &= \frac{T}{T_\infty}, & p^* &= \frac{p}{\rho_\infty V_\infty^2}, & e_t^* &= \frac{e_t}{V_\infty^2} \\ \bar{\bar{\tau}}^* &= \frac{\bar{\bar{\tau}} L}{\mu_\infty V_\infty}, & \mu^* &= \frac{\mu}{\mu_\infty}, & B_x^*, B_y^*, B_z^* &= \frac{B_x, B_y, B_z}{V_\infty \sqrt{\mu_e \rho_\infty}} \\ E_x^*, E_y^*, E_z^* &= \frac{E_x, E_y, E_z}{V_\infty^2 \sqrt{\mu_e \rho_\infty}}, & \mu_e^* &= \frac{\mu_e}{\mu_{e\infty}} = 1, & \sigma_e^* &= \frac{\sigma_e}{\sigma_{e\infty}} \end{aligned} \quad (5)$$

$$\mathbf{E}_v = \begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ u\tau_{xx} + v\tau_{xy} + w\tau_{xz} - q_x \end{bmatrix} \quad (9)$$

$$\mathbf{F}_v = \begin{bmatrix} 0 \\ \tau_{yx} \\ \tau_{yy} \\ \tau_{yz} \\ u\tau_{yx} + v\tau_{yy} + w\tau_{yz} - q_y \end{bmatrix} \quad (10)$$

$$\mathbf{G}_v = \begin{bmatrix} 0 \\ \tau_{zx} \\ \tau_{zy} \\ \tau_{zz} \\ u\tau_{zx} + v\tau_{zy} + w\tau_{zz} - q_z \end{bmatrix} \quad (11)$$

$$\mathbf{S}_{\text{MHD}} = Re_m \begin{bmatrix} 0 \\ B_z(E_y + wB_x - uB_z) - B_y(E_z + uB_y - vB_x) \\ B_x(E_z + uB_y - vB_x) - B_z(E_x + vB_z - wB_y) \\ B_y(E_x + vB_z - wB_y) - B_x(E_y + wB_x - uB_z) \\ E_x(E_x + vB_z - wB_y) + E_y(E_y + wB_x - uB_z) + E_z(E_z + uB_y - vB_x) \end{bmatrix} \quad (12)$$

For convenience, the asterisks are dropped in the following equations.

The governing equations written in vector form in a 3-D Cartesian coordinate system become

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{E}_i}{\partial x} + \frac{\partial \mathbf{F}_i}{\partial y} + \frac{\partial \mathbf{G}_i}{\partial z} = \frac{\partial \mathbf{E}_v}{\partial x} + \frac{\partial \mathbf{F}_v}{\partial y} + \frac{\partial \mathbf{G}_v}{\partial z} + \mathbf{S}_{\text{MHD}} \quad (6)$$

where  $\mathbf{S}_{\text{MHD}}$  contains all of the MHD effects. The flux vectors are given by

$$\mathbf{U} = [\rho, \rho u, \rho v, \rho w, \rho e_t]^T \quad (7)$$

$$\begin{aligned} \mathbf{E}_i &= \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ (\rho e_t + p)u \end{bmatrix}, & \mathbf{F}_i &= \begin{bmatrix} \rho v \\ \rho v^2 + p \\ \rho uv \\ \rho vw \\ (\rho e_t + p)v \end{bmatrix} \\ \mathbf{G}_i &= \begin{bmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho w^2 + p \\ (\rho e_t + p)w \end{bmatrix} \end{aligned} \quad (8)$$

where

$$\rho e_t = \rho \left[ e + \frac{1}{2}(u^2 + v^2 + w^2) \right] \quad (13)$$

and the nondimensional shear stresses and heat fluxes are defined in the usual manner [16].

The governing equations are transformed into computational space and written in a generalized coordinate system  $(\xi, \eta, \zeta)$  as

$$\frac{1}{J} \mathbf{U}_t + \tilde{\mathbf{E}}_\xi + \tilde{\mathbf{F}}_\eta + \tilde{\mathbf{G}}_\zeta = \frac{\mathbf{S}_{\text{MHD}}}{J} \quad (14)$$

where

$$\begin{aligned} \tilde{\mathbf{E}} &= \left( \frac{\xi_x}{J} \right) (\mathbf{E}_i - \mathbf{E}_v) + \left( \frac{\xi_y}{J} \right) (\mathbf{F}_i - \mathbf{F}_v) + \left( \frac{\xi_z}{J} \right) (\mathbf{G}_i - \mathbf{G}_v) \\ \tilde{\mathbf{F}} &= \left( \frac{\eta_x}{J} \right) (\mathbf{E}_i - \mathbf{E}_v) + \left( \frac{\eta_y}{J} \right) (\mathbf{F}_i - \mathbf{F}_v) + \left( \frac{\eta_z}{J} \right) (\mathbf{G}_i - \mathbf{G}_v) \\ \tilde{\mathbf{G}} &= \left( \frac{\zeta_x}{J} \right) (\mathbf{E}_i - \mathbf{E}_v) + \left( \frac{\zeta_y}{J} \right) (\mathbf{F}_i - \mathbf{F}_v) + \left( \frac{\zeta_z}{J} \right) (\mathbf{G}_i - \mathbf{G}_v) \end{aligned} \quad (15)$$

The governing equations are parabolized by dropping the time derivative term and the streamwise direction  $\xi$  viscous flow terms in the flux vectors. Equation (14) can then be rewritten as

$$\tilde{\mathbf{E}}_\xi + \tilde{\mathbf{F}}_\eta + \tilde{\mathbf{G}}_\zeta = \frac{\mathbf{S}_{\text{MHD}}}{J} \quad (16)$$

where

$$\begin{aligned}\tilde{\mathbf{E}} &= \left(\frac{\xi_x}{J}\right)\mathbf{E}_i + \left(\frac{\xi_y}{J}\right)\mathbf{F}_i + \left(\frac{\xi_z}{J}\right)\mathbf{G}_i \\ \tilde{\mathbf{F}} &= \left(\frac{\eta_x}{J}\right)(\mathbf{E}_i - \mathbf{E}'_v) + \left(\frac{\eta_y}{J}\right)(\mathbf{F}_i - \mathbf{F}'_v) + \left(\frac{\eta_z}{J}\right)(\mathbf{G}_i - \mathbf{G}'_v) \quad (17) \\ \tilde{\mathbf{G}} &= \left(\frac{\zeta_x}{J}\right)(\mathbf{E}_i - \mathbf{E}'_v) + \left(\frac{\zeta_y}{J}\right)(\mathbf{F}_i - \mathbf{F}'_v) + \left(\frac{\zeta_z}{J}\right)(\mathbf{G}_i - \mathbf{G}'_v)\end{aligned}$$

The primes in the preceding equations indicate that the streamwise viscous flow terms have been dropped.

For turbulent flows, the two-layer Baldwin–Lomax turbulence model [22] has been modified to account for MHD effects. Only the expression for turbulent viscosity in the inner layer is changed. This modification for MHD flows is due to Lykoudis [23].

To “close” the preceding system of PNS equations, relations between the thermodynamic variables are required, along with expressions for the transport properties  $\mu$  and  $k$ . For a perfect gas, the pressure is computed from the relation

$$p = (\tilde{\gamma} - 1)\rho e \quad (18)$$

where  $\tilde{\gamma} = \gamma_\infty$ , and the transport properties are computed using Sutherland’s formulas [16]. For equilibrium air computations,  $\tilde{\gamma}$  and all other thermodynamic and transport properties are obtained from the simplified curve fits of Srinivasan et al. [24,25]. These curve fits are based on clean-air chemistry without seeding. To simulate the effects of seeding on the flow, a constant electrical conductivity was applied in the powered portion of the flowfield. The value of the electrical conductivity was obtained from experimental measurements. The same procedure was also used for the perfect gas calculations. For nonequilibrium seeded-air computations, the thermodynamic and transport properties are determined using the procedures described in the next section. The electrical conductivity in this case is variable and is computed at each point using the species mole fractions obtained from the complete chemistry model.

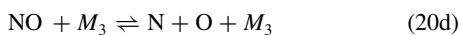
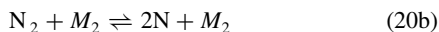
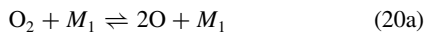
### Nonequilibrium Flow Equations

For nonequilibrium flows, the species continuity equations must be solved in addition to the magnetogasdynamic equations given previously. The magnetogasdynamic equations remain the same except for the additional term in the energy equation, which is due to the diffusion of the species. The nondimensional species continuity equations, expressed in 2-D transformed coordinates for a steady flow, are given by

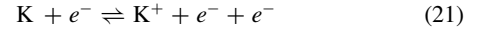
$$\begin{aligned}\rho u \left[ \frac{\partial c_s}{\partial \xi} + \left( \eta_x + \frac{v}{u} \eta_y \right) \frac{\partial c_s}{\partial \eta} \right] &= \dot{\omega}_s + \frac{\eta_y}{Re_\infty} \frac{\partial}{\partial \eta} \left( \beta_3 \rho D \eta_y \frac{\partial c_s}{\partial \eta} \right) \\ &+ \frac{\eta_x}{Re_\infty} \frac{\partial}{\partial \eta} \left( \beta_3 \rho D \eta_x \frac{\partial c_s}{\partial \eta} \right) \quad (s = 1, 2, \dots, n) \quad (19)\end{aligned}$$

where  $c_s$  is the mass fraction of species  $s$ ,  $\dot{\omega}_s$  is the nondimensional production term, and  $D$  is the nondimensional binary diffusion coefficient.

The chemistry model used in the present calculations is similar to the clean-air model of Blottner et al. [26] and Prabhu et al. [27]. It consists of molecular oxygen  $O_2$ , atomic oxygen  $O$ , molecular nitrogen  $N_2$ , atomic nitrogen  $N$ , nitric oxide  $NO$ , nitric oxide ion  $NO^+$ , and electrons  $e^-$ . The following reactions are considered between the constituent species.



where  $M_1$ ,  $M_2$ , and  $M_3$  are catalytic third bodies. The clean-air chemical model has seven species ( $n = 7$ ) and seven reactions ( $m = 7$ ). To simulate the seeded airflow in the MHD channel, the potassium-seeding reaction has been added to the preceding chemistry model. This reaction is the ionization of atomic potassium  $K$  and is given by the following equation:



Using the law of mass action, the nondimensional mass production rate of species  $s$  is

$$\begin{aligned}\dot{\omega}_s &= M_s \sum_{k=1}^m (v''_{k,s} - v'_{k,s}) \left[ K_{f,k}(T) \prod_{r=1}^{n_i} [\rho \gamma_r]^{v'_{k,r}} \right. \\ &\quad \left. - K_{b,k}(T) \prod_{r=1}^{n_i} [\rho \gamma_r]^{v''_{k,r}} \right] \quad (22)\end{aligned}$$

where  $\gamma_r$  is the nondimensional mole-mass ratio of the reactants,  $M_s$  is the molecular weight of species  $s$ ,  $v'_{k,s}$  and  $v''_{k,s}$  are the stoichiometric coefficients, and  $n_i$  is the number of reactants. Further details on the reaction rates and the thermodynamic and transport properties can be found in [26]. The electrical conductivity is determined from the species mole fractions, along with the temperature, density, and pressure of the gas, using the program of Park [21].

## Numerical Method

### Solution of PNS Equations

The governing PNS equations with MHD source terms have been incorporated into NASA’s UPS code [20]. These equations can be solved very efficiently using a single sweep of the flowfield for many applications. For cases in which upstream (elliptic) effects are important, the flowfield can be computed using multiple streamwise sweeps with either the IPNS [28], TIPNS [29], or FBIPNS [30] algorithms. This iterative process is continued until the solution is converged.

For the iterative PNS (IPNS) method, the  $\tilde{\mathbf{E}}$  vector is split using the Vigneron parameter [31]. This parameter does not need to be changed for the present low magnetic Reynolds number formulation. In the previous high magnetic Reynolds number code [17], it was necessary to modify the Vigneron parameter to account for MHD effects. After splitting, the  $\tilde{\mathbf{E}}$  vector can be written as

$$\tilde{\mathbf{E}} = \mathbf{E}^* + \mathbf{E}^p \quad (23)$$

where

$$\begin{aligned}\mathbf{E}^* &= \frac{\xi_x}{J} \begin{bmatrix} \rho u \\ \rho u^2 + \omega p \\ \rho uv \\ \rho uw \\ (\rho e_t + p)u \end{bmatrix} + \frac{\xi_y}{J} \begin{bmatrix} \rho v \\ \rho v^2 + \omega p \\ \rho vw \\ \rho vw \\ (\rho e_t + p)v \end{bmatrix} + \frac{\xi_z}{J} \begin{bmatrix} \rho w \\ \rho w^2 + \omega p \\ \rho vw \\ \rho vw \\ (\rho e_t + p)w \end{bmatrix} \\ \mathbf{E}^p &= \frac{\xi_x}{J} \begin{bmatrix} 0 \\ (1-\omega)p \\ 0 \\ 0 \\ 0 \end{bmatrix} + \frac{\xi_y}{J} \begin{bmatrix} 0 \\ 0 \\ (1-\omega)p \\ 0 \\ 0 \end{bmatrix} + \frac{\xi_z}{J} \begin{bmatrix} 0 \\ 0 \\ 0 \\ (1-\omega)p \\ 0 \end{bmatrix} \quad (24)\end{aligned}$$

The streamwise derivative of  $\tilde{\mathbf{E}}$  is then differenced using a backward

difference for  $\mathbf{E}^*$  and a forward difference for the elliptic portion  $\mathbf{E}^p$ :

$$\left(\frac{\partial \tilde{\mathbf{E}}}{\partial \xi}\right)_{i+1} = \frac{1}{\Delta \xi}[(\mathbf{E}_{i+1}^* - \mathbf{E}_i^*) + (\mathbf{E}_{i+2}^p - \mathbf{E}_{i+1}^p)] \quad (25)$$

where the subscript  $i + 1$  denotes the spatial index (in the  $\xi$  direction) for which the solution is currently being computed. The vectors  $\mathbf{E}_{i+1}^*$  and  $\mathbf{E}_{i+1}^p$  are then linearized in the following manner:

$$\begin{aligned} \mathbf{E}_{i+1}^* &= \mathbf{E}_i^* + \left(\frac{\partial \mathbf{E}^*}{\partial \mathbf{U}}\right)_i (\mathbf{U}_{i+1} - \mathbf{U}_i) \\ \mathbf{E}_{i+1}^p &= \mathbf{E}_i^p + \left(\frac{\partial \mathbf{E}^p}{\partial \mathbf{U}}\right)_i (\mathbf{U}_{i+1} - \mathbf{U}_i) \end{aligned} \quad (26)$$

The Jacobians can be represented by

$$A^* = \frac{\partial \mathbf{E}^*}{\partial \mathbf{U}} \quad A^p = \frac{\partial \mathbf{E}^p}{\partial \mathbf{U}} \quad (27)$$

After substituting the preceding linearizations into Eq. (25), the expression for the streamwise gradient of  $\tilde{\mathbf{E}}$  becomes

$$\left(\frac{\partial \tilde{\mathbf{E}}}{\partial \xi}\right)_{i+1} = \frac{1}{\Delta \xi}[(A_i^* - A_i^p)(\mathbf{U}_{i+1} - \mathbf{U}_i) + (\mathbf{E}_{i+2}^p - \mathbf{E}_i^p)] \quad (28)$$

The final discretized form of the fluid flow equations with MHD source terms is obtained by substituting Eq. (28) into Eq. (16) along with the linearized expressions for the fluxes in the crossflow plane. The final expression becomes

$$\left[\frac{1}{\Delta \xi}(A_i^* - A_i^p) + \frac{\partial}{\partial \eta} \left(\frac{\partial \tilde{\mathbf{F}}}{\partial \mathbf{U}}\right)_i + \frac{\partial}{\partial \zeta} \left(\frac{\partial \tilde{\mathbf{G}}}{\partial \mathbf{U}}\right)_i\right]^{k+1} (\Delta \mathbf{U}_i)^{k+1} = \mathbf{RHS} \quad (29)$$

where

$$(\Delta \mathbf{U}_i)^{k+1} = (\mathbf{U}_{i+1} - \mathbf{U}_i)^{k+1}$$

$$\begin{aligned} \mathbf{RHS} &= -\frac{1}{\Delta \xi}[(\mathbf{E}_{i+2}^p)^k - (\mathbf{E}_i^p)^{k+1}] - \left(\frac{\partial \tilde{\mathbf{F}}}{\partial \eta}\right)_i^{k+1} - \left(\frac{\partial \tilde{\mathbf{G}}}{\partial \zeta}\right)_i^{k+1} \\ &\quad + \left(\frac{\mathbf{S}_{\text{MHD}}}{J}\right)_i^{k+1} \end{aligned}$$

and the superscript  $k + 1$  denotes the current iteration (i.e., sweep) level. In the preceding equation,  $\mathbf{S}_{\text{MHD}}$  is treated explicitly, because it is evaluated using the velocity at station  $i$  ( $\mathbf{V}_i$ ). For most cases, this will not degrade the accuracy of the solution, because  $\Delta \xi$  is small and the velocity changes slowly. If this is not the case, a predictor-corrector procedure can be implemented whereby a predicted velocity at station  $i + 1$  ( $\mathbf{V}_{i+1}^{\sim}$ ) is first obtained using Eq. (29). The solution at station  $i + 1$  is then recomputed by evaluating  $\mathbf{S}_{\text{MHD}}$  with  $\mathbf{V}_{i+1}^{\sim}$ .

### Solution of Species Continuity Equations

For chemical nonequilibrium, the species continuity equations (19) must be solved, in addition to the magnetogasdynamic equations. The equations have been integrated using the loosely coupled approach of Tannehill et al. [32]. In this approach, the species continuity equations and magnetogasdynamic equations are solved separately. The coupling between the two sets of equations is then obtained in an approximate manner. The species continuity equations are modeled using a second-order-accurate, upwind-based, total variation diminishing (TVD) scheme for the convective terms and second-order-accurate central differences for the diffusion terms. The assumption of zero net charge of the gas is used to eliminate the electron mass conservation equation. In addition, the species continuity equation for the  $n$ th species is eliminated by using the requirement that the mass fractions must sum to unity. The term

representing the rate of production of species  $\dot{\omega}_s$  is treated as a source term and is lagged to the previous marching level.

The coupling between the fluids and the chemistry is performed in an approximate manner. First, a fluid step is taken from marching station  $i$  to  $i + 1$ , assuming frozen chemistry. Next, the fluid density and velocity at  $i + 1$  are used in the solution of the species continuity equations to obtain species mass fractions at  $i + 1$ . Finally, the species mass fractions, molecular weight of mixture, fluid density, and internal energy at  $i + 1$  are used to obtain the new temperature, pressure, specific enthalpy, and frozen specific heats at the  $i + 1$  marching station.

The temperature is obtained by performing a Newton-Raphson iteration of the following form:

$$T^{k+1} = T^k - \frac{g(T^k) - h}{g'(T^k)} \quad (30)$$

where

$$g(T) = \sum_{s=1}^n c_s h_s(T)$$

$$g'(T) = \sum_{s=1}^n c_s C_{p,s}(T)$$

and  $k$  is the index of iteration. The iterations are continued until

$$|T^{k+1} - T^k| \leq \epsilon$$

where  $\epsilon$  is a small positive quantity. Once the temperature is determined, the pressure can be computed using Dalton's law of partial pressures. Further details of this procedure can be found in [32,33].

### Numerical Results

The numerical calculation of the 3-D supersonic flow in the experimental MHD channel is now discussed. The flow in the nozzle section was computed using a combination of the OVERFLOW code [34] and the present PNS code (without MHD effects). For the OVERFLOW nozzle calculation, a highly stretched grid consisting of  $150 \times 80 \times 80$  grid points was used. The normal grid spacing at the wall was  $1.0 \times 10^{-5}$  m. For the PNS calculation of the flow in the remainder of the nozzle and the rest of the MHD channel, a highly stretched grid consisting of 90 points in both the  $y$  and  $z$  directions was used and the normal grid spacing at the wall was  $2.0 \times 10^{-6}$  m. As a consequence of flow symmetry, only one-fourth of the channel cross section was computed in the 3-D calculations. The experimental data confirm that the flow is reasonably symmetric in the horizontal direction, but no measurements were taken in the other direction.

The calculations were performed assuming turbulent flow throughout the MHD channel. The channel wall temperature was assumed to be isothermal, because quasi-steady flow conditions were maintained in the experiment for only about 1.2 ms. A schematic of the powered portion of the MHD channel, along with the directions of the applied magnetic and electric fields, is shown in Fig. 2. The values of the magnetic field  $B_z$ , and the electric field  $E_y$  were kept constant in the powered portion of the channel.

Three different chemistry models were used in this study to simulate the flow in the MHD channel. These were 1) perfect gas ( $\tilde{\gamma} = 1.25$ ), 2) equilibrium clean-air chemistry, and 3) nonequilibrium seeded-air chemistry. For the perfect gas and equilibrium clean-air calculations, the seeding was artificially introduced by adding a constant electrical conductivity. For the nonequilibrium seeded-air calculations, the electrical conductivity varied throughout the flowfield and was determined using the program of Park [21]. The seeding (as in the experiment) consisted of 1% (by mass) potassium. Two test cases corresponding to runs 15 and 16 of the NASA Ames experiments [2] were computed in this study and are now discussed.

**Test Case 1: NASA Ames MHD Run 15 ( $V_{cap} = 320$  volts)**

The dimensional flow parameters for this test case are

$$p_o = 9.10 \times 10^5 \text{ N/m}^2 \quad T_o = 5560 \text{ K} \quad T_w = 300 \text{ K}$$

$$\sigma_e = 130 \text{ mho/m (or variable)} \quad B_z = 0.0, 0.92 \text{ T}$$

$$E_y = 0, 3955, 5000 \text{ V/m}$$

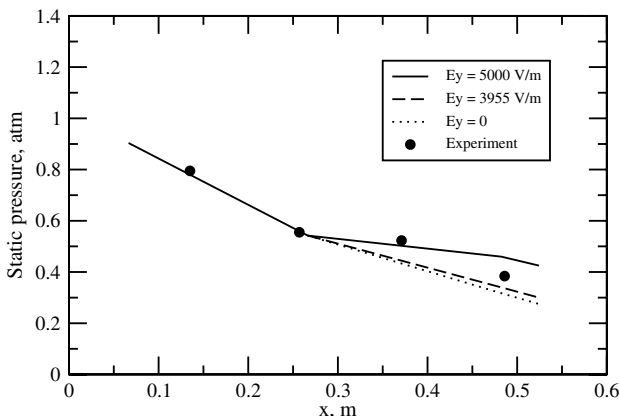
where the subscript  $o$  denotes total conditions at the nozzle entrance.

This case was computed using several different electric field strengths to properly simulate the experiment. In the experiment, the voltage applied to the electrodes was approximately 134 V for this case; however, due to the sheath voltage drop, the actual voltage applied to the flow is smaller than the electrode voltage. The voltage drop was measured for the central inviscid core flow and was approximately 67 V [2]. Because the boundary layer is computed in the numerical solution, the applied electric field must be approximately the voltage drop across the electrodes minus the sheath voltage drop. Unfortunately, it is not a trivial task to measure the sheath voltage drop. Therefore, several different electric field strengths were chosen in the numerical calculations so that the corresponding voltage drop across the electrodes would be between 67 and 134 V. The voltage drop of 67 V corresponds to  $E_y = 3955$  V/m and a voltage drop of 84.7 V corresponds to  $E_y = 5000$  V/m.

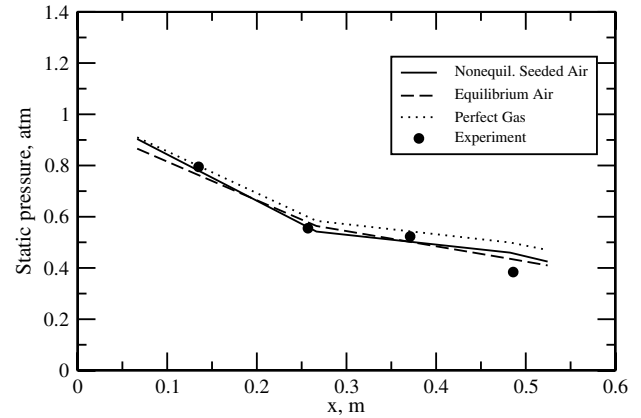
The computed streamwise variation of static pressure for the nonequilibrium seeded-air calculations is shown in Fig. 3 for the different electric field strengths. The  $x$  coordinate is measured from the nozzle throat. The pressure variation with no electric field or magnetic field is denoted by  $E_y = 0$ . The results for  $E_y = 5000$  V/m are in good agreement with the experiment. The numerical results show an increase in static pressure as the electric field strength is increased. The computed streamwise variation of static pressure for the different chemistry models is shown in Fig. 4 for  $E_y = 5000$  V/m. The nonequilibrium seeded-air calculation and the equilibrium clean-air calculation with artificial seeding give similar results. This indicates that reasonable results can be obtained, with much less computer time, using the latter approach.

For the nonequilibrium seeded-air computations, the electrical conductivity was not constant, but varied throughout the flowfield. The average conductivity (averaged over the channel cross section) at the center of the powered portion of the channel (electrode pair 10) was found to be 130 mho/m for  $E_y = 5000$  V/m. This value of conductivity is within the range determined in the experiments (94–140 mho/m). A constant value of 130 mho/m was used for the perfect gas and equilibrium air computations.

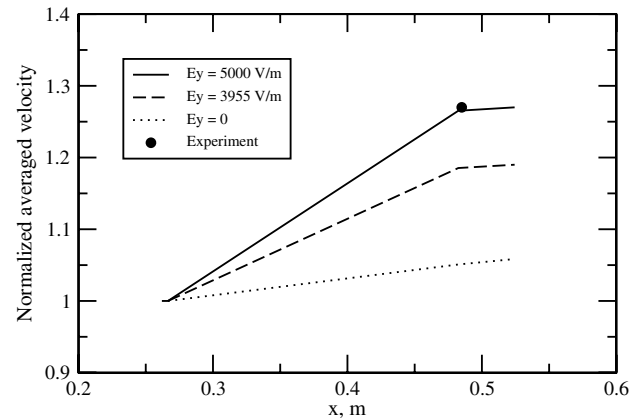
The computed streamwise variation of averaged velocity for the nonequilibrium seeded-air calculations is shown in Fig. 5 for the powered portion of the channel. The velocities are averaged across the channel cross section and normalized using the entrance velocity. In the experiment, the velocities were obtained by measuring the



**Fig. 3 Streamwise variation of static pressure for run 15 (non-equilibrium seeded-air model).**



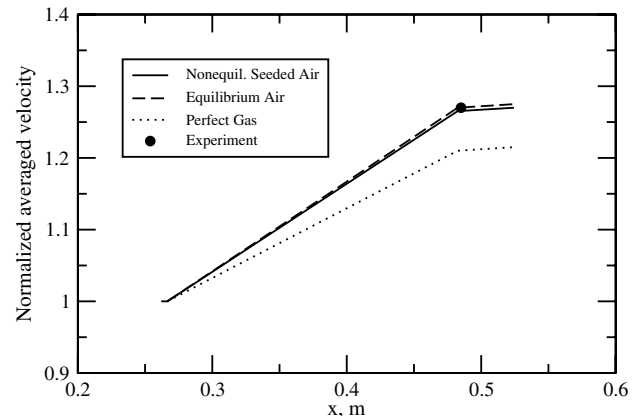
**Fig. 4 Streamwise variation of static pressure for run 15 ( $E_y = 5000$  V/m).**



**Fig. 5 Streamwise variation of averaged velocity for run 15 (non-equilibrium seeded-air model).**

voltage generated by the flow at the last electrode pair (pair 19), which is unpowered. This procedure inherently involves an averaging of the velocity profile. The experimental results were normalized using the velocities obtained with no electric field ( $E_y = 0$ ) but with a magnetic field ( $B_z = 0.92$  T). The numerical results indicate an increase in the averaged velocity of about 27% with  $E_y = 5000$  V/m, and this agrees exactly with the experimental value of 27%.

The computed streamwise variation of averaged velocity for the different chemistry models is shown in Fig. 6 for  $E_y = 5000$  V/m. Both the equilibrium air and the nonequilibrium seeded-air chemistry models give similar results. The centerline variation of static temperature for the nonequilibrium seeded-air model is given



**Fig. 6 Streamwise variation of averaged velocity for run 15 ( $E_y = 5000$  V/m).**

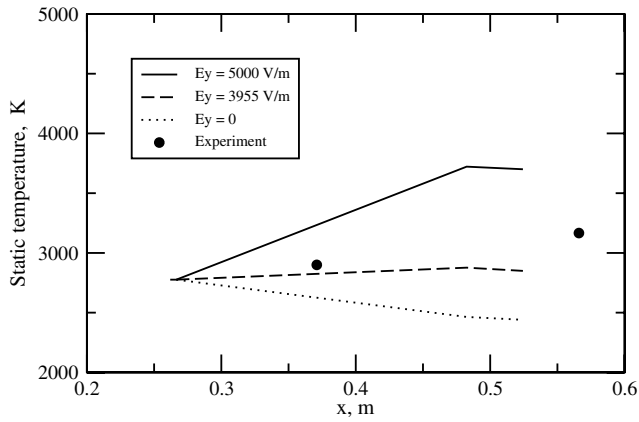


Fig. 7 Streamwise variation of static temperature for run 15 (nonequilibrium seeded-air model).

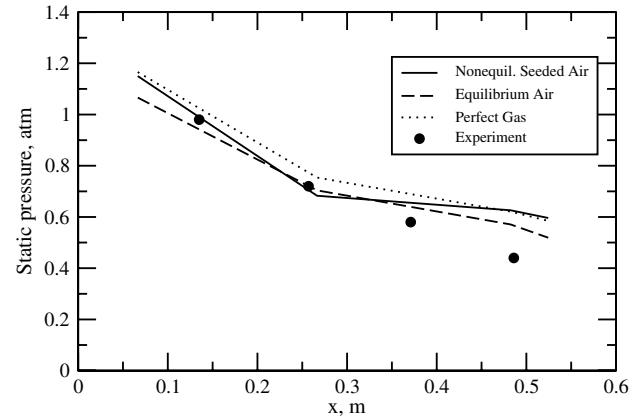


Fig. 9 Streamwise variation of static pressure for run 16 ( $E_y = 6000$  V/m).

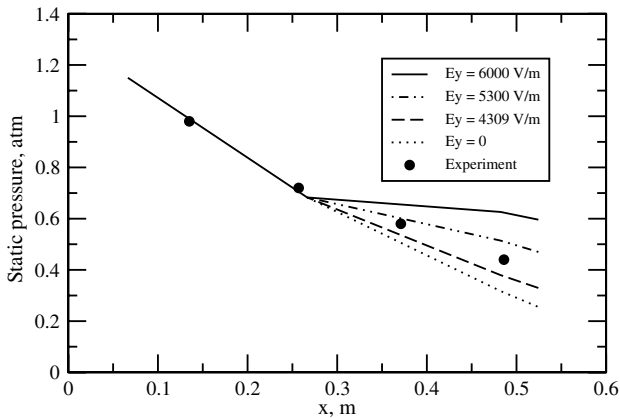


Fig. 8 Streamwise variation of static pressure for run 16 (non-equilibrium seeded-air model).

in Fig. 7 for the different electric field strengths. The computed static temperatures are in reasonable agreement with the experimental values.

#### Test Case 2: NASA Ames MHD Run 16 ( $V_{cap} = 380$ volts)

The dimensional flow parameters for this test case are

$$p_o = 9.92 \times 10^5 \text{ N/m}^2 \quad T_o = 5560 \text{ K} \quad T_w = 300 \text{ K}$$

$$\sigma_e = 130 \text{ mho/m (or variable)} \quad B_z = 0.0, 0.92 \text{ T}$$

$$E_y = 0, 4309, 5300, 6000 \text{ V/m}$$

This test case was also computed using several different electric field strengths to properly simulate the experiment. The electric field strength of 4309 V/m corresponds to the voltage drop measured in the central inviscid core flow. The computed streamwise variation of static pressure for the nonequilibrium seeded-air calculations is shown in Fig. 8 for the different electric field strengths. The experimental pressure variation agrees closest to the numerical result with an electric field strength of 5300 V/m. The computed streamwise variation of static pressure for the different chemistry models is shown in Fig. 9 for  $E_y = 6000$  V/m. The different chemistry models produce somewhat similar results.

The averaged electrical conductivity at the center of the powered portion of the channel (electrode pair 10) for the nonequilibrium seeded-air calculations was found to be 142 mho/m for  $E_y = 5300$  V/m. This value is at the upper end of the range determined in the experiments (120–144 mho/m). A constant value of 130 mho/m was used in the perfect gas and equilibrium air computations.

The computed streamwise variation of averaged velocity for the nonequilibrium seeded-air calculations is shown in Fig. 10. The

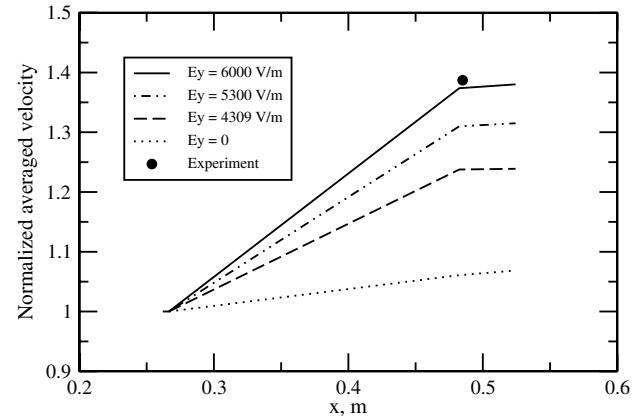


Fig. 10 Streamwise variation of averaged velocity for run 16 (nonequilibrium seeded-air model).

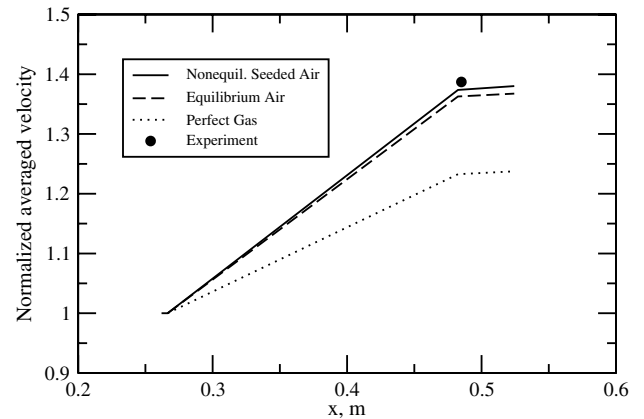
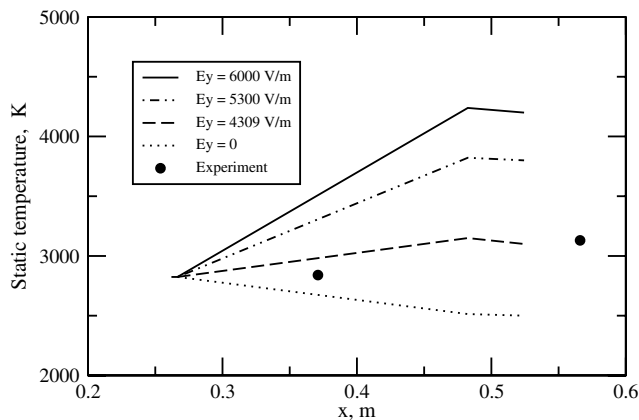


Fig. 11 Streamwise variation of averaged velocity for run 16 ( $E_y = 6000$  V/m).

numerical results indicate an increase in the averaged velocity of about 38% for  $E_y = 6000$  V/m, and this agrees closely with the experimental value of 39%. The computed streamwise variation of averaged velocity for the different chemistry models is shown in Fig. 11 for  $E_y = 6000$  V/m. Once again, the equilibrium air and the nonequilibrium seeded-air models give similar results. The centerline variation of static temperature for the equilibrium model is shown in Fig. 12 for the different electric field strengths, along with the experimental values.

## Conclusions

In this study, a new 3-D parabolized Navier–Stokes algorithm with nonequilibrium seeded-air capability has been developed to



**Fig. 12 Streamwise variation of static temperature for run 16 (nonequilibrium seeded-air model).**

efficiently compute MHD flows in the low magnetic Reynolds number regime. The new algorithm has been used to compute the flow in the NASA Ames experimental MHD channel for runs 15 and 16. The numerical results are in good agreement with the experimental results. It has been shown that reasonable results can be obtained at a fraction of the computer cost by using an equilibrium clean-air chemistry model along with artificial seeding (i.e., assuming a constant value of electrical conductivity that agrees with the experiment).

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